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Spatial Correlations in Bounded Nonequilibrium Fluid Systems

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We study the influence of boundaries on the equal-time thermal correlations in a three-dimensional fluid maintained under a constant temperature gradient. Within the confines of the model for an idealized fluid bounded by two infinite, parallel walls, we show that it is crucial to retain the unbounded spatial components in the problem so that the solutions approach meaningful results as we move the walls infinitely far apart. In addition, we consider a composite system by including the dynamics of the "walls," and we investigate the conditions for the relevant physical parameters under which the details of wall dynamics may be neglected by employing the simple boundary condition $\delta T = 0$.

KEY WORDS: Correlation functions; nonequilibrium steady states; boundary effects.

1. INTRODUCTION

The study of fluctuations in nonequilibrium stationary states of fluid systems has received considerable attention in the literature.[1-8] Prior studies of systems in the steady state range from phenomenological[7] to microscopic treatments based on kinetic theory or mode-coupling theory.[4-6] Despite the numerous theoretical approaches employed, the different techniques generally yielded similar results. In addition to the breaking of time-reversal symmetry, one obtains nonlocal long-ranged correlations of the hydrodynamic variables in a noncritical state. These predictions have in fact been confirmed by recent experiments performed by Law, Sengers, *et al.*[9-12]

Most of the previous studies have considered hydrodynamic steady states in the limit of a large system where boundary effects may be

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neglected. Recently, there has been renewed interest in the problem of non-equilibrium fluctuations in finite systems bounded by solid walls.[13-22] Using the fluctuating hydrodynamics formalism, several papers have addressed the spatial correlations of the various conserved densities (number, momentum, and energy) of bounded fluids subject to nonequilibrium constraints.

We shall focus in this paper on the static temperature autocorrelation function for a fluid maintained under a constant temperature gradient. In previous work, the temperature–temperature correlation function has been predicted to be long-ranged and encompassing the entire system.[15,17,18] The absence of an intrinsic characteristic correlation length persisted even in the limit of a large system of size L. Our results for this problem are different.

This paper is organized as follows: A brief review of the results of Rubí *et al.*[15] and Garcia *et al.*[18] is presented in Section 2. Using the fluctuating hydrodynamics formalism, we obtain the temperature correlator for a three-dimensional fluid in Section 3 and show that it reduces to the infinite domain solution as $L \to \infty$. Unlike the previous researchers, we retain the unbounded spatial dimensions in the derivation. In Section 4, we study the dynamics of a composite system by coupling the thermal modes of the walls to system variables. We show the conditions under which the problem can be simplified with the simple boundary condition used in the derivation of Section 3. In Section 5, we present our conclusions.

2. REVIEW OF THE PROBLEM

In this section we present a brief summary of the results of Rubí *et al.*[15] and Garcia *et al.*[18] for completeness. The fluid is bounded by two parallel plates at $z = 0$, L and is infinite in the x, y directions. The boundaries are rigid, impermeable, and held in contact with heat reservoirs at T_0 and T_L, respectively, where $T_L > T_0$. The two plates are assumed to be perfectly heat conducting; thus, the temperature of the fluid at $z = 0$, L is equal to the temperature of the reservoirs. Following previous researchers, we impose the restriction that the thermal expansion coefficient vanishes. This condition decouples the energy equation from the density and velocity equations, thereby vastly simplifying the analysis. Furthermore, the transport and thermodynamic coefficients of the model fluid are taken to be constants.

The stationary solution to the heat diffusion equation

$$\frac{\partial}{\partial t} T(\mathbf{r}, t) = \alpha_T \nabla^2 T(\mathbf{r}, t) \tag{1}$$

is

$$T(\mathbf{r}) = T(z) = T_0 + \mathbf{r} \cdot \nabla T \tag{2}$$

where α_T is the thermal diffusivity and $\nabla T = (\mathbf{e}_z(T_L - T_0)/L$. The equation of motion for fluctuations about the steady state has an additional contribution to the heat flux modeled by a stochastic source:

$$\frac{\partial}{\partial t} T(\mathbf{r}, t) = \alpha_T \nabla^2 T(\mathbf{r}, t) - \frac{1}{\rho C_p} \nabla \cdot \mathbf{J}(\mathbf{r}, t) \tag{3}$$

where ρ is the mass density and C_p is the heat capacity per unit mass at constant pressure. The random part of the heat flux is assumed to be a Gaussian white noise whose correlation function is given by

$$\langle \mathbf{J}(\mathbf{r}, t)\, \mathbf{J}(\mathbf{r}', t') \rangle = 2k_B \lambda T^2(\mathbf{r})\, \delta(\mathbf{r} - \mathbf{r}')\, \delta(t - t')\, \mathbf{I} \tag{4}$$

where k_B is Boltzmann's constant, λ is the thermal conductivity of the fluid, $\mathbf{I}$ is the identity matrix, and $\langle \cdot \rangle$ denotes a steady-state average.

The model is further simplfied by taking the parallel spatial Fourier transform in the x, y plane. Then the fluctuating hydrodynamic variable δT is reduced to

$$\delta T(z; \mathbf{k}_{\|}, t) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, e^{-i(k_x x + k_y y)}\, \delta T(\mathbf{r}, t) \tag{5}$$

where $k_{\|}^2 = k_x^2 + k_y^2$.

Defining

$$C(z, z') = \frac{\langle \delta T(z; \mathbf{k}_{\|} = 0, t)\, \delta T(z'; \mathbf{k}_{\|} = 0, t) \rangle}{(2\pi)^2\, \delta(\mathbf{k}_{\|} + \mathbf{k}'_{\|})} \tag{6}$$

one gets for the static correlation function[15,18]

$$C(z, z') = k_B (\rho C_p)^{-1}\, T^2(z)\, \delta(z - z') + f(z, z') \tag{7}$$

where the first term is simply the local equilibrium contribution, and the second term is the nonequilibrium contribution given by

$$f(z, z') = k_B (\rho C_p)^{-1} \frac{(\nabla T)^2}{L} \{\theta(z' - z)\, z(L - z') + \theta(z - z')\, z'(L - z)\} \tag{8}$$

where θ is the unit step function. The boundary condition used corresponds to $\delta T = 0$ at the boundaries (perfectly conducting plates). As a result, the delta-function contribution to the temperature–temperature correlation is expandable in a Fourier sine series, i.e.,

$$\delta(z-z')=\frac{2}{L}\sum_{n=0}^{\infty}\sin\left(\frac{n\pi z}{L}\right)\sin\left(\frac{n\pi z'}{L}\right) \tag{9}$$

We note in passing that the procedure described in this section inappropriately reduces the description to a one-dimensional system.

3. REANALYSIS OF THE PROBLEM: PART I

The nonequilibrium contribution to the static correlation function predicted in refs. 15 and 18 is long-ranged and encompasses the entire system. The result would therefore suggest that the thermal fluctuations at any two given points are always positively correlated regardless of the size of the system. For large L and ∇T constant, however, we should expect that the solution would exhibit an intrinsic length scale and decay as in an infinite system.

We shall attempt to resolve this paradox in two ways. First, we rederive the static temperature correlator in a three-dimensional system and show that the solution behaves properly in the limit of a large system. Second, in the next section, we justify the simple boundary condition $\delta T=0$ at $z=0$, L by considering a composite system consisting of reservoirs plus the system of interest. By analyzing the thermal fluctuations in the composite system, we arrive at a sequence of characteristic relaxation times which allows us to predict a bound on the size of the system where boundary effects are important.

By combining the equations of motion for $\delta T(\mathbf{r}, t)$ and $\delta T(\mathbf{r}', t)$ and using relations (4) and

$$\begin{aligned}\langle \delta T(\mathbf{r}, t)\, \mathbf{J}(\mathbf{r}', t)\rangle &= -\frac{k_B\lambda}{\rho C_p}\nabla_r\cdot\{T^2(\mathbf{r})\,\delta(\mathbf{r}-\mathbf{r}')\,\mathbf{I}\}\\ &= -\frac{k_B\lambda}{\rho C_p}\nabla_r\{T^2(\mathbf{r})\,\delta(\mathbf{r}-\mathbf{r}')\}\end{aligned} \tag{10}$$

it is easy to show that[17,18]

$$\begin{aligned}\frac{\partial}{\partial t}\langle \delta T(\mathbf{r}, t)\,\delta T(\mathbf{r}', t)\rangle &= \alpha_T(\nabla_r^2+\nabla_{r'}^2)\langle \delta T(\mathbf{r}, t)\,\delta T(\mathbf{r}', t)\rangle\\ &\quad + 2\frac{k_B\lambda}{(\rho C_p)^2}\nabla_r\cdot\nabla_{r'}\{T^2(\mathbf{r})\,\delta(\mathbf{r}-\mathbf{r}')\}\end{aligned} \tag{11}$$

Using $T(\mathbf{r}) = T_0 + \mathbf{r} \cdot \nabla T = T(z)$, one obtains for the correlator at steady state

$$(\nabla_r^2 + \nabla_{r'}^2)\overline{\langle \delta T(\mathbf{r}, t)\, \delta T(\mathbf{r}', t)\rangle} + 2\,\frac{(\nabla T)^2\, k_\mathrm{B}}{\rho C_p}\,\delta(\mathbf{r} - \mathbf{r}') = 0 \tag{12}$$

where

$$\overline{\langle \delta T(\mathbf{r}, t)\, \delta T(\mathbf{r}', t)\rangle} \equiv \langle \delta T(\mathbf{r}, t)\, \delta T(\mathbf{r}', t)\rangle - \frac{k_\mathrm{B} T^2(\mathbf{r})}{\rho C_p}\,\delta(\mathbf{r} - \mathbf{r}') \tag{13}$$

We shall omit the time dependence of the correlator, since we are dealing with the steady state. Examination of Eq. (12) reveals that it is essentially the same as the Poisson equation for the electric field generated by a point charge. Thus, the temperature–temperature correlator defined by (13) may be interpreted as the potential at $\mathbf{r}$ corresponding to a source $(\nabla T)^2\, k_\mathrm{B}/\rho C_p$ at $\mathbf{r}'$. For the case of an infinite system, the potential is a function of $|\mathbf{r} - \mathbf{r}'|$ only:

$$\overline{\langle \delta T(\mathbf{r})\, \delta T(\mathbf{r}')\rangle} = \frac{(\nabla T)^2\, k_\mathrm{B}}{\rho C_p}\left(\frac{1}{4\pi\, |\mathbf{r} - \mathbf{r}'|}\right) \tag{14}$$

This long-range behavior decaying as $|\mathbf{r} - \mathbf{r}'|^{-1}$ has also been predicted for the density–momentum correlation in a infinite fluid subject to a temperature gradient.[7]

Using the boundary condition $\delta T = 0$ for $z = 0$, L, it is straightforward to show that the correlation function for the bounded fluid is given by[23]

$$\overline{\langle \delta T(\mathbf{r})\, \delta T(\mathbf{r}')\rangle}$$

$$= \frac{(\nabla T)^2\, k_\mathrm{B}}{\rho C_p}\left(\frac{1}{4\pi\, |\mathbf{r} - \mathbf{r}'|} + \sum_{n=-\infty}^{\infty}{}^{\{n \neq 0\}} \frac{1}{4\pi\, |\mathbf{r} - \mathbf{r}'_\mathbf{n}|} - \sum_{n=-\infty}^{\infty} \frac{1}{4\pi\, |\mathbf{r} - \mathbf{r}''_\mathbf{n}|}\right) \tag{15}$$

where $\mathbf{r}'_\mathbf{n} = \mathbf{e}_x x' + \mathbf{e}_y y' + \mathbf{e}_z(2nL + z')$ and $\mathbf{r}''_\mathbf{n} = \mathbf{e}_x x' + \mathbf{e}_y y' + \mathbf{e}_z(2nL - z')$. Note that in Eq. (15) we have explicitly separated the term corresponding to the infinite-dimension solution from the first summation. To be specific, we shall consider the case where $z' = L/2$. The infinite series given in Eq. (15) does not converge rapidly unless we have the condition

$$|\mathbf{r} - \mathbf{r}'| \ll L \tag{16}$$

When this condition is satisfied, the temperature correlation function is given by the infinite-domain solution plus a small correction term. Thus, the influence of the boundaries on the fluctuations in the bulk of the fluid vanishes as the plates move infinitely far apart, which is the behavior we expect from physical intuition. A similar result was obtained by Spohn[22] for the nonequilibrium correlations of a stochastic lattice gas.

4. REANALYSIS OF THE PROBLEM: PART II

In this section we shall motivate the simplified boundary condition $\delta T = 0$ by including the dynamics of the heat bath in our model. Following Procaccia *et al.*,[1] we shall consider an isolated composite system with three compartments, two of which are the reservoirs separated by the system of interest. The boundaries of the system of interest are located at $-L$ and L. Again, we shall consider only the thermal modes of each compartment. Thus, the corresponding heat diffusion equation is given by

$$\frac{\partial}{\partial t} T(\mathbf{r}, t) = \alpha_T(\mathbf{r})\, \nabla^2 T(\mathbf{r}, t) \tag{17}$$

where the thermal diffusivity $\alpha_T(\mathbf{r}) = \alpha_T(z)$ is given by

$$\alpha_T(\mathbf{r}) = \begin{cases} \alpha_T^B & \text{for} \quad -L_B \leqslant z \leqslant -L \\ \alpha_T^S & \text{for} \quad -L \leqslant z \leqslant L \\ \alpha_T^B & \text{for} \quad L \leqslant z \leqslant L_B \end{cases}$$

The boundaries between the compartments are rigid and impermeable, but heat conducting. By imposing the boundary conditions that $T(\mathbf{r}, t)$ and the energy flux are continuous and the initial condition

$$T(\mathbf{r}, 0) = T(z, 0) = \begin{cases} T_{-L} & \text{for} \quad -L_B \leqslant z \leqslant -L \\ T_S & \text{for} \quad -L \leqslant z \leqslant L \\ T_L & \text{for} \quad L \leqslant z \leqslant L_B \end{cases}$$

it can be shown that for an intermediate time scale the temperature is well approximated by[1]

$$T(\mathbf{r}, t) = T(z, t) = \begin{cases} T_{-L} & \text{for} \quad -L_B \leqslant z \leqslant -L \\ T_{-L} + \dfrac{T_L - T_{-L}}{2L}(z + L) & \text{for} \quad -L \leqslant z \leqslant L \\ T_L & \text{for} \quad L \leqslant z \leqslant L_B \end{cases}$$

Specifically, for the time interval

$$\frac{L^2}{\alpha_T^S} < t < \frac{(L_B - L)^2}{\alpha_T^B} \tag{18}$$

and the condition

$$\alpha_T^B/\alpha_T^S \to \infty \tag{19}$$

the middle compartment has the usual steady-state temperature distribution. Thus, in order to maintain this quasi-nonequilibrium steady state for a long time, we choose highly conducting reservoirs of large extent so that the conditions (19) and $(L_B - L)^2/\alpha_T^B \to \infty$ are satisfied.

Within the time interval (18) where a steady state is maintained, we will in the following analyze the corresponding thermal fluctuations. The equation of motion for $\delta T(\mathbf{r}, t)$ is given by

$$\frac{\partial}{\partial t}\,\delta T(\mathbf{r}, t) = \alpha_T(\mathbf{r})\,\nabla^2\,\delta T(\mathbf{r}, t) - \alpha_T(\mathbf{r})\,\lambda^{-1}(\mathbf{r})\,\nabla\cdot\mathbf{J}(\mathbf{r}, t) \tag{20}$$

where $\alpha_T(\mathbf{r})$ was given previously in this section and $\alpha_T\lambda^{-1} = (\rho C_p)^{-1}$ is $\alpha_T^B\lambda_B^{-1}$ for the bath and $\alpha_T^S\lambda_S^{-1}$ for the system. Using the relation

$$\tilde{A}(z; \mathbf{k}_{\|}, w) = \int_{-\infty}^{\infty} A(\mathbf{r}, t)\exp(iwt - i\mathbf{k}_{\|}\cdot\mathbf{r}_{\|})\,d\mathbf{r}_{\|}\,dt \tag{21}$$

where A is an arbitrary dynamical variable, we reduce Eq. (20) to an ordinary differential equation in the variable z:

$$\left[i\tilde{w} + \alpha_T(z)\frac{\partial^2}{\partial z^2}\right]\delta\tilde{T} = \alpha_T(z)\,\lambda^{-1}(z)\left(\frac{\partial}{\partial z}\tilde{\mathbf{J}}_z + i\mathbf{k}_{\|}\cdot\tilde{\mathbf{J}}_{\|}\right) \tag{22}$$

with $\tilde{w} = w + i\alpha_T(z)\,k_{\|}^2$. Whenever confusion can arise, we will denote $\tilde{w}_B$ as $w + i\alpha_T^B k_{\|}^2$ and $\tilde{w}_S$ as $w + i\alpha_T^S k_{\|}^2$. The solution to the preceding equation can be expressed in terms of a Green's function:

$$\delta\tilde{T}(z; \mathbf{k}_{\|}, w) = \alpha_T(z)\,\lambda^{-1}(z)\int_{-L_B}^{L_B} dz'\,G(z', z; \mathbf{k}_{\|}, w)\left(\frac{\partial}{\partial z'}\tilde{\mathbf{J}}_{z'} + i\mathbf{k}_{\|}\cdot\tilde{\mathbf{J}}_{\|}\right) \tag{23}$$

where the Green's function satisfies the equation

$$\left[i\tilde{w} + \alpha_T(z)\frac{\partial^2}{\partial z^2}\right]G(z, z'; \mathbf{k}_{\|}, w) = \delta(z - z') \tag{24}$$

and is subject to the homogeneous boundary conditions corresponding to (e.g., ref. 24)

$$\lambda_B \frac{\partial}{\partial z} G(z, z'; \mathbf{k}_{\|}, w)\bigg|_{z=\pm L_B} = 0 \tag{25}$$

$$G(-L-0, z'; \mathbf{k}_{\|}, w) = G(-L+0, z'; \mathbf{k}_{\|}, w) \tag{26}$$

$$\lambda_B \frac{\partial}{\partial z} G(z, z'; \mathbf{k}_{\|}, w)\bigg|_{z=-L-0} = \lambda_S \frac{\partial}{\partial z} G(z, z'; \mathbf{k}_{\|}, w)\bigg|_{z=-L+0} \tag{27}$$

$$G(L-0, z'; \mathbf{k}_{\|}, w) = G(L+0, z'; \mathbf{k}_{\|}, w) \tag{28}$$

$$\lambda_S \frac{\partial}{\partial z} G(z, z'; \mathbf{k}_{\|}, w)\bigg|_{z=L-0} = \lambda_B \frac{\partial}{\partial z} G(z, z'; \mathbf{k}_{\|}, w)\bigg|_{z=L+0} \tag{29}$$

Since we are interested in fluctuations in the middle compartment only, we shall quote the explicit form for the Green's function $G(z, z'; \mathbf{k}_{\|}, w)$ for $z, z' \in (-L, +L)$:

$$\begin{aligned} G(z, z'; \mathbf{k}_{\|}, w) = &-\frac{\gamma_S}{\alpha_T^S}\left\{\sinh\left(\frac{L+z_<}{\gamma_S}\right) + \frac{\lambda_S}{\gamma_S\delta}\cosh\left(\frac{L+z_<}{\gamma_S}\right)\right\} \\ &\times\left\{\sinh\left(\frac{L-z_>}{\gamma_S}\right) + \frac{\lambda_S}{\gamma_S\delta}\cosh\left(\frac{L-z_>}{\gamma_S}\right)\right\} \\ &\times\left[\left\{\sinh\left(\frac{2L}{\gamma_S}\right) + \frac{\lambda_S}{\gamma_S\delta}\cosh\left(\frac{2L}{\gamma_S}\right)\right\}\right. \\ &\left.+\frac{\lambda_S}{\gamma_S\delta}\left\{\cosh\left(\frac{2L}{\gamma_S}\right) + \frac{\lambda_S}{\gamma_S\delta}\sinh\left(\frac{2L}{\gamma_S}\right)\right\}\right]^{-1} \end{aligned} \tag{30}$$

where $z_< = \min(z, z')$, $z_> = \max(z, z')$, $\gamma_S = (i\alpha_T^S/\tilde{w}_S)^{1/2}$, $\gamma_B = (i\alpha_T^B/\tilde{w}_B)^{1/2}$, and

$$\delta = \frac{\lambda_B}{\gamma_B}\tanh\left(\frac{L_B - L}{\gamma_B}\right)$$

Note that alternatively G in Eq. (30) satisfies the effective boundary condition

$$\lambda_S \frac{\partial}{\partial z} G(z, z'; \mathbf{k}_{\|}, w) \pm \delta G(z, z'; \mathbf{k}_{\|}, w) = 0$$

$$\text{for} \quad z = \pm L \quad \text{and} \quad z' \in (-L, +L) \tag{31}$$

where δ is a frequency-dependent temperature-slip coefficient of the boundary.[25] In effect, we have included all the dynamics of the heat bath into a single surface transport coefficient. We remark that Eq. (30) for the Green's function with a constant δ coefficient was obtained by Pagonabarraga *et al.*[16]

Using standard manipulations with Green's functions, it is straightforward to show that the expression for the temperature correlation function between two points within the middle compartment is given by

$$\begin{aligned}
&\frac{\langle \delta T(z; \mathbf{k}_{\|}, w)\, \delta T(z'; \mathbf{k}'_{\|}, w)\rangle}{2k_{\mathrm{B}}(\alpha_T^S \lambda_S^{-1})^2\, \delta(w+w')\, \delta(\mathbf{k}_{\|}+\mathbf{k}'_{\|})} \\
&\quad = \frac{1}{2}\left\{ -\frac{1}{\alpha_T^S \lambda_S^{-1}} \left[G(z', z)\, T^2(z') + G^*(z, z')\, T^2(z) \right] \right. \\
&\qquad + i\frac{\lambda_S(\nabla T)^2}{w} \left[G(z', z) - G^*(z, z') \right] \\
&\qquad - \lambda_S \left[G(z_1, z)\, G^*(z_1, z') \frac{\partial}{\partial z_1} T^2(z_1) \right] \Bigg|_{z_1 = \pm L} \\
&\qquad - i\frac{\lambda_S(\nabla T)^2 \alpha_T^S}{w} \left[G(z_1, z) \frac{\partial}{\partial z_1} G^*(z_1, z') \right. \\
&\qquad \left.\left. - G^*(z_1, z') \frac{\partial}{\partial z_1} G(z_1, z) \right] \Bigg|_{z_1 = \pm L} \right\}
\end{aligned} \tag{32}$$

where we have suppressed the w and $\mathbf{k}_{\|}$ dependence in $G(z, z')$ for notational simplicity, the asterisk denotes replacing w by $-w$, and $\nabla T = (T_L - T_{-L})/2L$. The preceding expression can be simplified since we will consider the limits $(L_B - L)^2/\alpha_T^B \to \infty$ and $\alpha_T^B/\alpha_T^S \to \infty$. Under these constraints, the combination $\lambda_S/\gamma_S\delta$ that appears in the Green's function in Eq. (30) becomes

$$\frac{\lambda_S}{\gamma_S \delta} = \left(\frac{\lambda_S}{\lambda_B}\right)\left(\frac{\gamma_B}{\gamma_S}\right) \coth\left(\frac{L_B - L}{\gamma_B}\right) \to 0 \tag{33}$$

When condition (33) is satisfied, the boundary condition for G, Eq. (31), reduces to

$$G(\pm L, z'; \mathbf{k}_{\|}, w) = 0 \tag{34}$$

or equivalently $\delta T = 0$ at the boundaries. The previous conclusion is perhaps not surprising since, intuitively, we expect a perfectly conducting bath to dissipate thermal fluctuations infinitely fast at the boundaries. We may thus neglect the last two terms in Eq. (32) and insert for G

$$G(z, z'; \mathbf{k}_{\|}, w) = G(z, z', \tilde{w}_S)$$
$$= -\frac{\gamma_S}{\alpha_T^S} \frac{\sinh((L+z_<)/\gamma_S)\sinh((L-z_>)/\gamma_S)}{\sinh(2L/\gamma_S)} \tag{35}$$

By examining Eq. (35), we can infer two time scales, $(L+z_<)^2/\alpha_T^S$ and $(L-z_>)^2/\alpha_T^S$, corresponding to the time it takes for a thermal fluctuation generated at $z_<$ (or $z_>$) to reach $-L$ (or L). Physically, we expect that if this thermal diffusion time is much longer than the typical hydrodynamic relaxation time, we may neglect the boundary effects. Indeed, if we take the limit $L^2/\alpha_T^S \to \infty$ (but $[L^2/(L_B-L)^2]\, \alpha_T^B/\alpha_T^S \ll 1$) and with z and z' not at the boundaries, G in Eq. (35) reduces to the Green's function for an infinite system.

For a finite system, we expand Eq. (35) in a Fourier series:

$$G(z, z', \tilde{w}_S) = \sum_{n=-\infty}^{\infty} G_n(z', \tilde{w}_S)\, e^{i(n\pi z/L)} \tag{36}$$

In order to calculate the equal-time averages for the temperature correlator, we must perform an integration over w and w' of Eq. (32). It is easy to show, using the series expansion for G, that the first term in Eq. (32) will generate the local equilibrium contribution proportional to $T^2(z)\,\delta(z-z')$. The remaining nonlocal contribution is given by the expression

$$\overline{\langle \delta T(\mathbf{r})\, \delta T(\mathbf{r}')\rangle}$$
$$= i\frac{\lambda_S(\nabla T)^2 k_B(\alpha_T^S \lambda_S^{-1})^2}{(2\pi)^3} \int_{-\infty}^{\infty} d\mathbf{k}_{\|} \int_{-\infty}^{\infty} dw \exp[i\mathbf{k}_{\|}\cdot(\mathbf{r}_{\|}-\mathbf{r}'_{\|})]$$
$$\times \sum_{n=-\infty}^{\infty} \left\{\frac{G_n(z', \tilde{w}_S) - G_n^*(z', \tilde{w}_S)}{w}\right\} \exp i\frac{n\pi z}{L} \tag{37}$$

This result is formally equivalent to Eq. (15) except for the change in the boundaries of the system from 0, L to $-L$, L. In the limit of large L, suffice it to say that the Fourier series converts to an integral with $n\pi/L = k_z$, yielding as a result the three-dimensional Fourier transform of $1/k^2$, which is proportional to $|\mathbf{r}-\mathbf{r}'|^{-1}$, as expected.

5. CONCLUSION

In this paper we have examined the influence of boundaries on the thermal correlations of a three-dimensional fluid system. We have restricted our analysis to an idealized system in which the thermal modes are uncoupled from the other conserved densities. Furthermore, the assumption that the thermodynamic and transport coefficients of the fluid are constants restricts the length scale for which we probe the system to be smaller than a characteristic macroscopic length. We emphasize that for this particular model it is important to retain the parallel (i.e., unbounded) spatial components in the problem so that the temperature correlator behaves properly as we let the boundaries move infinitely far apart.

Ultimately, the influence of boundaries on bulk dynamics depends on the length and time scales for which we probe the physical system. As mentioned previously in Section 4, one can roughly estimate the importance of boundaries by comparing the thermal diffusion time L^2/α_T^S to the characteristic relaxation time one is probing, say τ_{probe}. For the case $L^2/\alpha_T^S \gg \tau_{\text{probe}}$, one should be able to neglect the boundaries for correlations within the fluid system. For concreteness, we can choose parameters corresponding to a typical light scattering experiment on water at 283 K, which has a thermal diffusivity of $1.38 \times 10^{-3}\,\text{cm}^2\,\text{sec}^{-1}$. For this system, if the characteristic time scale one is probing is of the order of 10^{-7} sec, the previous argument would suggest that boundary effects can be neglected for thermal bulk dynamics if L is greater than 10^{-5} cm.

Our present results differ markedly from those obtained in refs. 15, 17, and 18. We have considered a three-dimensional system and have properly treated the spatial components parallel to the walls across which heat is transferred. Even though there is a temperature gradient in the z direction only, this is not a one-dimensional system, which the previous results effectively describe.

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